Exponential Asymptotics and Resurgence

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Exponential asymptototics has been a very active area of research in the last 3 decades. It started with fundamental work by Berry, Écalle and Kruskal. Small exponentials are usually responsible for the divergence of asymptotic series. Resurgence enables the divergent tails to be decoded to yield these exponentials. Including these small exponentials leads to exponentially-improved asymptotics at several levels: hyperasymptotics. Via resurgence we are now able to compute the so-called connection coefficients, or Stokes multipliers. I will demonstrate the latest progress of exponential asymptotics for ODEs and PDEs.

These new techniques give us also better representations for the remainders in the asymptotic expansions and this has led recently to much sharper error bounds for the asymptotic expansions of many special functions.

The smoothing of the Stokes phenomenon via an error function was introduced by Berry in 1989. I will discuss the higher order Stokes phenomenon and its smoothing via combinations of error functions.

In the final part I will discuss second order linear ODEs with a large parameter. I will show that in some cases the corresponding Stokes multiplier can vanish completely, and this will give us reflectionlessness potentials.